Warm-up!

A) What transformation turns the graph of $f(x) = \log$ into the graph of $g(x) = \log_2(8x)$? $= \log_2 8 + \log_2 x = 3 + \log_2 x$

Horizontal dilation by a factor of $\frac{1}{8}$ = Vertical translation 3 units

B) What transformation turns the graph of into the graph of $h(x) = \log_2(\sqrt[3]{x})$? $= \log_2\left(x^{\frac{1}{3}}\right) = \frac{1}{3}\log_2 x$

Vertical dilation by a factor of $\frac{1}{3}$

C) What transformation turns the graph of into the graph of $k(x) = \log_{\frac{1}{2}}(x)$?

Reflection over x-axis

$$= \frac{\log_2 x}{\log_2\left(\frac{1}{2}\right)} = \frac{\log_2 x}{-1} = -\log_2 x$$

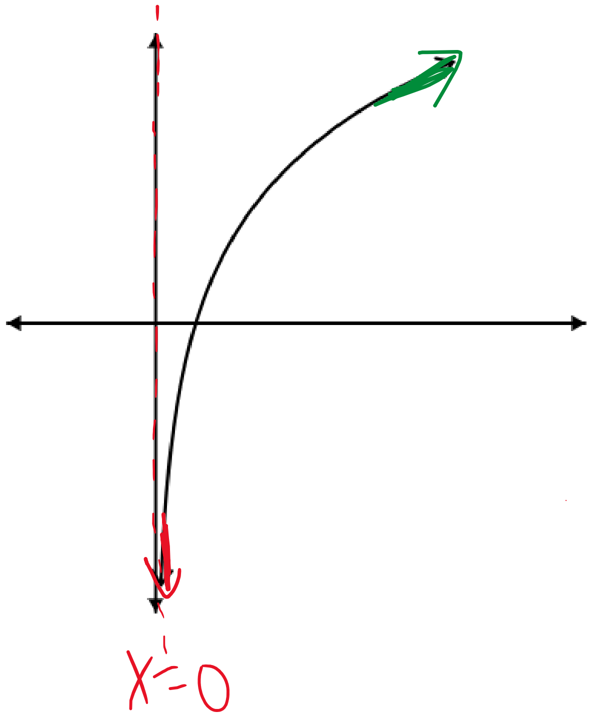
Log Functions Day 3: (2.11) Graphs of Log Functions

Homework: Logarithmic Functions and Their Graphs

Quiz: Class After Next. Goes up to the material from today (including the material from today)

Graphs of Logarithmic Functions

$$f(x) = a \log_b x$$
$a > 0$ and $b > 1$



Domain: $(0, \infty)$

Range: $(-\infty, \infty)$

$\lim_{x \to \infty} f(x) =$ ∞

$\lim_{x \to 0^+} f(x) =$ $-\infty$

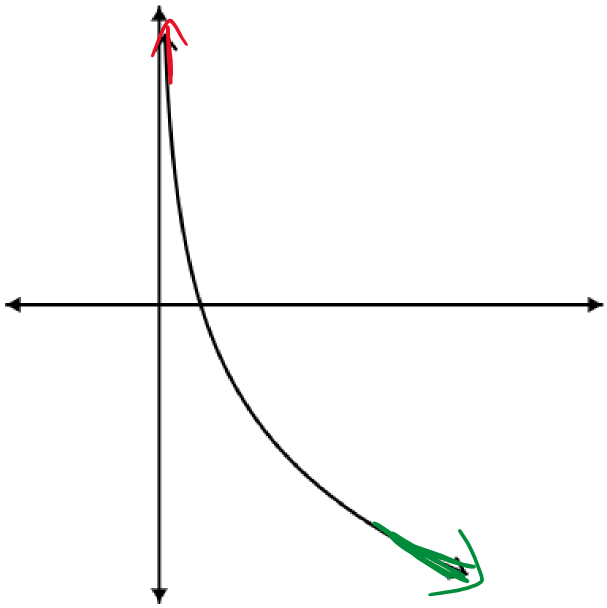
f(x) Increasing vs Decreasing

f(x) CCU vs CCD

What happens if $a < 0$

Graphs of Logarithmic Functions

$$f(x) = a \log_b x$$
$a < 0$ and $b > 1$



Domain: $(0, \infty)$

Range: $(-\infty, \infty)$

$\lim_{x \to \infty} f(x) =$ $-\infty$

$\lim_{x \to 0^+} f(x) =$ ∞

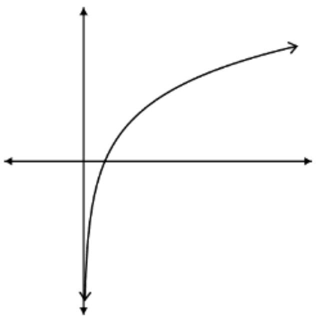
Increasing vs Decreasing

CCU vs CCD

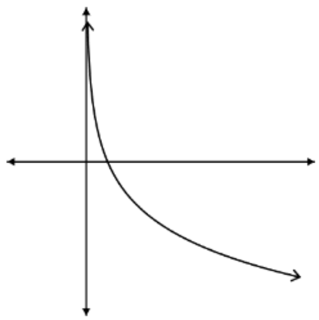
What happens if $0 < b < 1$

If the base of the log is $0 < b < 1$, then it flips the graph vertically, just like $a < 0$

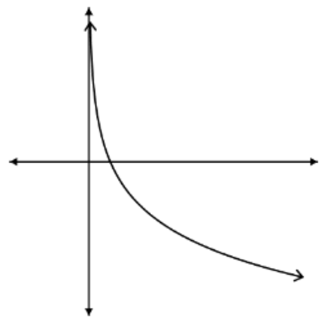
$y = \log_4(x)$



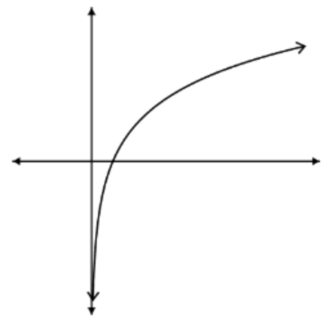
$y = \log_{\frac{1}{4}}(x)$



$y = -\log_4(x)$



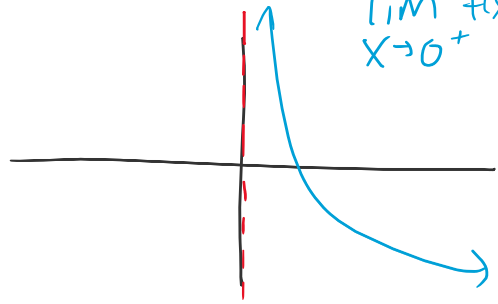
$y = -\log_{\frac{1}{4}}(x)$



Describe the vertical asymptote and end behavior using limits

A) $f(x) = -4\ln(x)$

$x=0$



$\lim_{x \to 0^+} f(x) = \infty$

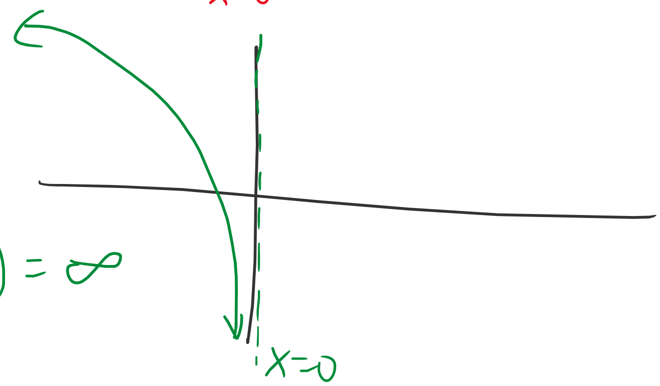
$\lim_{x \to \infty} f(x) = -\infty$

$x=0$

B) $g(x) = 6\log_2(-x)$

$-x=0$

$x=0$



$x=0$

$\lim_{x \to -\infty} g(x) = \infty$

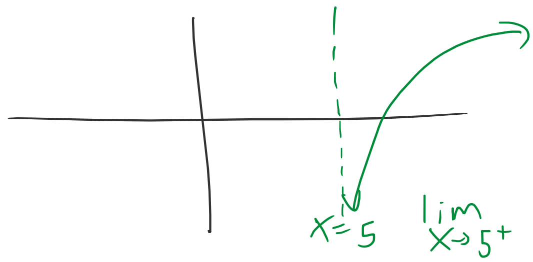
$\lim_{x \to 0^-} g(x) = -\infty$

Describe the vertical asymptote and end behavior using limits

C) $f(x) = \log(x-5)+2$

$x-5=0$

$x=5$



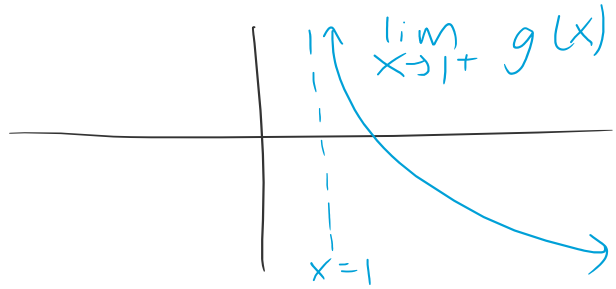
$\lim_{x\to\infty} f(x) = \infty$

$\lim_{x\to 5^+} f(x) = -\infty$

D) $g(x) = -\log_4(x-1)$

$x-1=0$

$x=1$



$\lim_{x\to 1^+} g(x) = \infty$

$\lim_{x\to\infty} g(x) = -\infty$

Find the Domain

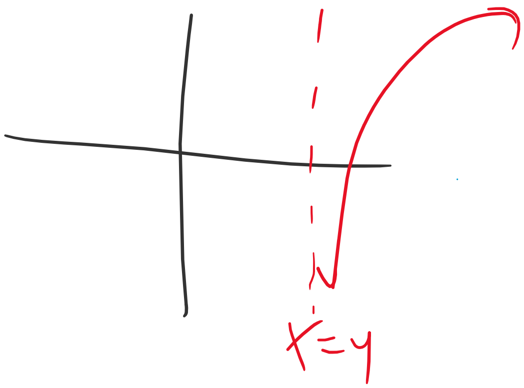
A) $f(x) = \log_2(x - 4)$

Graphically

Algebraically

$x - 4 > 0$

$x > 4$



Domain: $x > 4$

$(4, \infty)$

Find the Domain

B) $g(x) = 3\log_5(x+6) + 1$

$x + 6 > 0$

$x > -6$

$(-6, \infty)$

C) $h(x) = \frac{1}{2}\ln(-x) - 5$

$-x > 0$

$x < 0$

$(-\infty, 0)$

Let $g(x) = \log_3(3x - 1) + 4$

Find the domain.

$3x - 1 > 0$

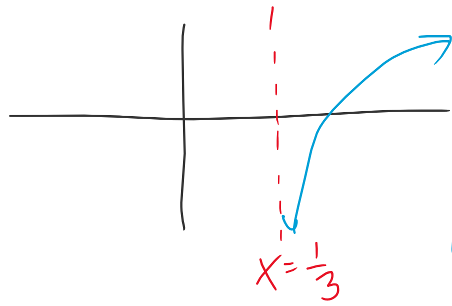
$3x > 1$

$x > \frac{1}{3}$

Does this function increase or decrease?

Is this function concave up or concave down?

Describe the end behavior and vertical asymptote using limits.



$\lim_{x \to \infty} g(x) = \infty$

$\lim_{x \to \frac{1}{3}^+} g(x) = -\infty$

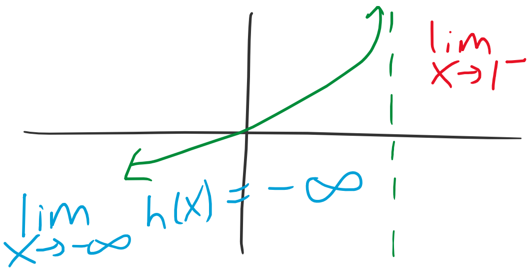
Find the range.

$(-\infty, \infty)$

Let $h(x) = -2\log_3(1 - x) + 10$

Find the domain.

$1 - x > 0$
$-x > -1$
$x < 1$



$\lim_{x \to 1^-} h(x) = \infty$

$\lim_{x \to -\infty} h(x) = -\infty$

$x = 1$

Does this function increase or decrease?

Is this function concave up or concave down?

Describe the end behavior and vertical asymptote using limits.

Find the range. $(-\infty, \infty)$

$f(x)$ is of the form $f(x) = a\log_b(x - c) + d$

How can we tell $f(x)$ is of this form?

+16 +8 +4 +2

x	13	29	37	41	43
$f(x)$	6	10	14	18	22

+4 +4 +4 +4

The first differences in the input values are changing proportionally ($\times \frac{1}{2}$).

The outputs are equally spaced.

$g(x) = 2x + 1$

x	0	1	2	3	4
$g(x)$	1	3	5	7	9

+2 +2 +2 +2

Justifying our choice for "Look at this table. What type of function is it?" questions.

Statement	Justification
f is best modelled by a linear function...	...since consecutive equally spaced input values correspond with output values with constant first differences
f is best modelled by a quadratic function...	...since consecutive equally spaced input values correspond with output values with constant second differences
f is best modelled by a cubic function...	...since consecutive equally spaced input values correspond with output values with constant third differences
f is best modelled by an exponential function...	...since consecutive equally spaced input values correspond with output values that change proportionally (in the case of an exponential function that underwent a vertical translation, you will instead note that consecutive equally spaced input values correspond with output values with **first differences** that change proportionally)
f is best modelled by a logarithmic function...	...since consecutive equally spaced output values correspond with input values that change proportionally (in the case of a logarithmic function that underwent a horizontal translation, you will instead note that consecutive equally spaced output values correspond with input values with **first differences** that change proportionally)

Some Examples of Tables of Values for Different Function Types

		$+8$	$+8$	$+8$	$+8$	$+8$
x	-6	2	10	18	26	34
$f(x)$	20	13	9	8	10	15
First Differences of Output Values		-7	-4	-1	$+2$	$+5$
Second Differences of Output Values		$+3$	$+3$	$+3$	$+3$	

Quadratic. Equally spaced inputs correspond with outputs with a constant second difference $(+3)$.

		$\times\frac{1}{2}$	$\times\frac{1}{2}$	$\times\frac{1}{2}$	$\times\frac{1}{2}$	$\times\frac{1}{2}$
x	160	80	40	20	10	5
$g(x)$	-3	3	9	15	21	27
		$+6$	$+6$	$+6$	$+6$	$+6$

		$+5$	$+5$	$+5$	$+5$	$+5$
x	-10	-5	0	5	10	15
$h(x)$	-50	31	85	121	145	161
First Differences of Output Values		$+81$	$+54$	$+36$	$+24$	$+16$
		$\times\frac{2}{3}$	$\times\frac{2}{3}$	$\times\frac{2}{3}$	$\times\frac{2}{3}$	

$g(x)$ and $h(x)$ are logarithmic functions. Find the value of k and p

$+54 \xrightarrow{\times \frac{1}{3}} +18 \xrightarrow{\times \frac{1}{3}} +6 \quad +2$

x	-36	18	36	$k = 42$	44
$g(x)$	5	10	15	20	25

$+5 \quad +5 \quad +5 \quad +5$

$\frac{18}{54} = \frac{1}{3}$

$? \quad +24 \quad +60 \xrightarrow{\cdot \frac{5}{2}} +150 \xrightarrow{\cdot \frac{5}{2}} +375$

x	$p = 1$	25	85	235	610
$h(x)$	-4	-2	0	2	4

$+2 \quad +2 \quad +2 \quad +2$

$\frac{150}{60} = \frac{15}{6} = \frac{5}{2} =$

$\frac{375}{150} = \frac{75}{30} = \frac{25}{10} = \frac{5}{2}$

$? \cdot \frac{5}{2} = 60$

$? = 24$

Find the equation of the vertical asymptote of each of the following functions.

A) $f(x) = 2\log_3(2x - 8) - 4$

B) $f(x) = -4\log_2\left(\frac{1}{2}x + 3\right) + 1$

C) $f(x) = \frac{1}{3}\ln(5 - x) + 8$

D) $f(x) = -8\log_6(-x) + 10$

Which one of the above functions decreases at an increasing rate?

Rewrite each as a single logarithm.

$-\ln(6x^3) + 2\ln(3x^4)$

$-\log_2(10x^7) - 2\log_2(x) + 3$

$$\log_2(x) - \log_8(x) - 2\log_2(2x-1)$$

$$\log_2(x) - \frac{\log_2 x}{\log_2 8} - 2\log_2(2x-1)$$

$$\log_2(x) - \frac{\log_2 x}{3} - 2\log_2(2x-1)$$

$$\log_2(x) - \frac{1}{3}\log_2 x - 2\log_2(2x-1)$$

$$\log_2\left(\frac{(x)}{(x)^{\frac{1}{3}}\,(2x-1)^2}\right)$$

$$\log_2\left(\frac{x^{2/3}}{(2x-1)^2}\right)$$